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ΑΠΑΝΤΗΣΕΙΣ ΣΤΗΝ ΤΡΙΓΩΝΟΜΕΤΡΙΑ
(ΦΥΣΙΚΟΜΑΘΗΜΑΤΙΚΟΣ ΚΥΚΛΟΣ - ΓΕΩΠΟΝΟΔΑΣΟΛΟΓΙΚΟΣ ΚΥΚΛΟΣ)
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Ζήτημα 1ο

$$\begin{cases} \alpha\cdot\sigma\upsilon\nu^2x + \beta\cdot\eta\mu^2x + \gamma\cdot\eta\mu x\cdot\sigma\upsilon\nu x + \delta = 0 \\ \alpha'\cdot\sigma\upsilon\nu^2x + \beta'\cdot\eta\mu^2x + \gamma'\cdot\eta\mu x\cdot\sigma\upsilon\nu x + \delta' = 0 \end{cases} \Leftrightarrow$$

$$\begin{cases} \alpha\cdot\dfrac{1+\sigma\upsilon\nu 2x}{2} + \beta\cdot\dfrac{1-\sigma\upsilon\nu 2x}{2} + \gamma\cdot\dfrac{\eta\mu 2x}{2} + \delta = 0 \\ \alpha'\cdot\dfrac{1+\sigma\upsilon\nu 2x}{2} + \beta'\cdot\dfrac{1-\sigma\upsilon\nu 2x}{2} + \gamma'\cdot\dfrac{\eta\mu 2x}{2} + \delta' = 0 \end{cases} \Leftrightarrow$$

$$\begin{cases} \alpha + \alpha\cdot\sigma\upsilon\nu 2x + \beta - \beta\cdot\sigma\upsilon\nu 2x + \gamma\cdot\eta\mu 2x + 2\delta = 0 \\ \alpha' + \alpha'\cdot\sigma\upsilon\nu 2x + \beta' - \beta'\cdot\sigma\upsilon\nu 2x + \gamma'\cdot\eta\mu 2x + 2\delta' = 0 \end{cases} \Leftrightarrow$$

$$\begin{cases} (\alpha-\beta)\cdot\sigma\upsilon\nu 2x + \gamma\cdot\eta\mu 2x = -(\alpha+\beta+2\delta) \\ (\alpha'-\beta')\cdot\sigma\upsilon\nu 2x + \gamma'\cdot\eta\mu 2x = -(\alpha'+\beta'+2\delta') \end{cases}$$

$$D = \begin{vmatrix} \alpha-\beta & \gamma \\ \alpha'-\beta' & \gamma' \end{vmatrix} = \boxed{\gamma'(\alpha-\beta) - \gamma(\alpha'-\beta') \neq 0 \quad \mathbf{(1)}}$$

$$D_1 = \begin{vmatrix} -(\alpha+\beta+2\delta) & \gamma \\ -(\alpha'+\beta'+2\delta') & \gamma' \end{vmatrix} = \gamma(\alpha'+\beta'+2\delta') - \gamma'(\alpha+\beta+2\delta)$$

$$D_2 = \begin{vmatrix} \alpha-\beta & -(\alpha+\beta+2\delta) \\ \alpha'-\beta' & -(\alpha'+\beta'+2\delta') \end{vmatrix} = (\alpha'-\beta')(\alpha+\beta+2\delta) - (\alpha-\beta)(\alpha'+\beta'+2\delta')$$

$$\sigma\upsilon\nu 2x = \frac{D_1}{D} = \frac{\gamma(\alpha'+\beta'+2\delta') - \gamma'(\alpha+\beta+2\delta)}{\gamma'(\alpha-\beta) - \gamma(\alpha'-\beta')}$$

$$\eta\mu 2x = \frac{D_2}{D} = \frac{(\alpha'-\beta')(\alpha+\beta+2\delta) - (\alpha-\beta)(\alpha'+\beta'+2\delta')}{\gamma'(\alpha-\beta) - \gamma(\alpha'-\beta')}$$

Είναι $\eta\mu^2 2x + \sigma\upsilon\nu^2 2x = 1 \Leftrightarrow$

$$\left[\frac{(\alpha'-\beta')(\alpha+\beta+2\delta) - (\alpha-\beta)(\alpha'+\beta'+2\delta')}{\gamma'(\alpha-\beta) - \gamma(\alpha'-\beta')}\right]^2 + \left[\frac{\gamma(\alpha'+\beta'+2\delta') - \gamma'(\alpha+\beta+2\delta)}{\gamma'(\alpha-\beta) - \gamma(\alpha'-\beta')}\right]^2 = 1 \Leftrightarrow$$

$$\frac{[(\alpha'-\beta')(\alpha+\beta+2\delta) - (\alpha-\beta)(\alpha'+\beta'+2\delta')]^2}{[\gamma'(\alpha-\beta) - \gamma(\alpha'-\beta')]^2} + \frac{[\gamma(\alpha'+\beta'+2\delta') - \gamma'(\alpha+\beta+2\delta)]^2}{[\gamma'(\alpha-\beta) - \gamma(\alpha'-\beta')]^2} = 1 \Leftrightarrow$$

$$\frac{[(\alpha'-\beta')(\alpha+\beta+2\delta) - (\alpha-\beta)(\alpha'+\beta'+2\delta')]^2 + [\gamma(\alpha'+\beta'+2\delta') - \gamma'(\alpha+\beta+2\delta)]^2}{[\gamma'(\alpha-\beta) - \gamma(\alpha'-\beta')]^2} = 1 \Leftrightarrow$$

$$\boxed{[(\alpha'-\beta')(\alpha+\beta+2\delta)-(\alpha-\beta)(\alpha'+\beta'+2\delta')]^2 + [\gamma(\alpha'+\beta'+2\delta')-\gamma'(\alpha+\beta+2\delta)]^2 = [\gamma'(\alpha-\beta)-\gamma(\alpha'-\beta')]^2 \quad (2)}$$

Επομένως οι ζητούμενες σχέσεις είναι οι (1) και (2).

Ζήτημα 2^{o}

$$\alpha\cdot\sigma\upsilon\nu\frac{A}{2}\cdot\eta\mu\frac{B-\Gamma}{2}+\beta\cdot\sigma\upsilon\nu\frac{B}{2}\cdot\eta\mu\frac{A-\Gamma}{2}+\gamma\cdot\eta\mu\frac{\gamma}{2}\cdot\sigma\upsilon\nu\frac{A-B}{2}>0 \Leftrightarrow$$

$$\alpha\cdot\eta\mu\frac{B+\Gamma}{2}\cdot\eta\mu\frac{B-\Gamma}{2}+\beta\cdot\eta\mu\frac{A+\Gamma}{2}\cdot\eta\mu\frac{A-\Gamma}{2}+\gamma\cdot\sigma\upsilon\nu\frac{A+B}{2}\cdot\sigma\upsilon\nu\frac{A-B}{2}>0 \Leftrightarrow$$

$$\frac{1}{2}\alpha\cdot(\sigma\upsilon\nu\Gamma-\sigma\upsilon\nu B)+\frac{1}{2}\beta\cdot(\sigma\upsilon\nu\Gamma-\sigma\upsilon\nu A)+\frac{1}{2}\gamma\cdot(\sigma\upsilon\nu B+\sigma\upsilon\nu A)>0 \Leftrightarrow$$

$$\frac{1}{2}R\cdot\eta\mu A\cdot(\sigma\upsilon\nu\Gamma-\sigma\upsilon\nu B)+\frac{1}{2}R\cdot\eta\mu B\cdot(\sigma\upsilon\nu\Gamma-\sigma\upsilon\nu A)+\frac{1}{2}R\cdot\eta\mu\Gamma\cdot(\sigma\upsilon\nu B+\sigma\upsilon\nu A)>0 \Leftrightarrow$$

$$\frac{1}{2}\cdot R\cdot(\eta\mu A\sigma\upsilon\nu\Gamma-\eta\mu A\sigma\upsilon\nu B+\eta\mu B\sigma\upsilon\nu\Gamma-\eta\mu B\sigma\upsilon\nu A+\eta\mu\Gamma\sigma\upsilon\nu B+\eta\mu\Gamma\sigma\upsilon\nu A)>0 \Leftrightarrow$$

$$\frac{1}{2}\cdot R\cdot[(\eta\mu B\sigma\upsilon\nu\Gamma+\eta\mu\Gamma\sigma\upsilon\nu B)+(\eta\mu A\sigma\upsilon\nu\Gamma+\eta\mu\Gamma\sigma\upsilon\nu A)-(\eta\mu A\sigma\upsilon\nu B+\eta\mu B\sigma\upsilon\nu A)]>0 \Leftrightarrow$$

$$\frac{1}{2}\cdot R\cdot[\eta\mu(B+\Gamma)+\eta\mu(A+\Gamma)-\eta\mu(A+B)]>0 \Leftrightarrow$$

$$\frac{1}{2}\cdot R\cdot(\eta\mu A+\eta\mu B-\eta\mu\Gamma)>0 \Leftrightarrow$$

$$\frac{1}{2}\cdot(R\cdot\eta\mu A+R\cdot\eta\mu B-R\cdot\eta\mu\Gamma)>0 \Leftrightarrow$$

$\frac{1}{2}\cdot(\alpha+\beta-\gamma)>0$ που ισχύει διότι $\alpha+\beta>\gamma$ (τριγωνική ανισότητα)

Ζήτημα 3^{o}

$$\begin{cases}\eta\mu x+\eta\mu y=\alpha \quad (1)\\ \sigma\upsilon\nu x+\sigma\upsilon\nu y=\beta \quad (2)\\ \varepsilon\varphi x+\varepsilon\varphi y=\gamma \quad (3)\end{cases}$$

$$(1) \Leftrightarrow 2\eta\mu\frac{x+y}{2}\sigma\upsilon\nu\frac{x-y}{2}=\alpha \Leftrightarrow \eta\mu\frac{x+y}{2}\sigma\upsilon\nu\frac{x-y}{2}=\frac{\alpha}{2} \quad (4)$$

$$(2) \Leftrightarrow 2\sigma\upsilon\nu\frac{x+y}{2}\sigma\upsilon\nu\frac{x-y}{2}=\beta \Leftrightarrow \sigma\upsilon\nu\frac{x+y}{2}\sigma\upsilon\nu\frac{x-y}{2}=\frac{\beta}{2} \quad (5)$$

$$(4),(5) \overset{(:)}{\Rightarrow} \varepsilon\varphi\frac{x+y}{2}=\frac{\alpha}{\beta} \quad (6)$$

$$(3) \Leftrightarrow \frac{\eta\mu x}{\sigma\upsilon\nu x}+\frac{\eta\mu y}{\sigma\upsilon\nu y}=\gamma \Leftrightarrow \frac{\eta\mu x\sigma\upsilon\nu y+\eta\mu y\sigma\upsilon\nu x}{\sigma\upsilon\nu x\sigma\upsilon\nu y}=\gamma \Leftrightarrow$$

$$\frac{\eta\mu(x+y)}{\sigma\upsilon\nu x\sigma\upsilon\nu y}=\gamma \Leftrightarrow \frac{2\eta\mu(x+y)}{2\sigma\upsilon\nu x\sigma\upsilon\nu y}=\gamma \Leftrightarrow \frac{2\eta\mu(x+y)}{\sigma\upsilon\nu(x+y)+\sigma\upsilon\nu(x-y)}=\gamma \quad (7)$$

$$ημ(x+y)=\frac{2εφ\frac{x+y}{2}}{1+εφ^2\frac{x+y}{2}}\overset{(6)}{=}\frac{2\frac{α}{β}}{1+\frac{α^2}{β^2}}=\frac{\frac{2α}{β}}{\frac{α^2+β^2}{β^2}}=\frac{2αβ}{α^2+β^2}\quad(8)$$

$$συν(x+y)=\frac{1-εφ^2\frac{x+y}{2}}{1+εφ^2\frac{x+y}{2}}\overset{(6)}{=}\frac{1-\frac{α^2}{β^2}}{1+\frac{α^2}{β^2}}=\frac{\frac{β^2-α^2}{β^2}}{\frac{α^2+β^2}{β^2}}=\frac{β^2-α^2}{α^2+β^2}\quad(9)$$

$$(1),(2)\ \Rightarrow \begin{cases}(ημx+ημy)^2=α^2\\(συνx+συνy)^2=β^2\end{cases}\Leftrightarrow\begin{cases}1+2ημxημy=α^2\\1+2συνxσυνy=β^2\end{cases}\Leftrightarrow$$

$$\begin{cases}2ημxημy=α^2-1\\2συνxσυνy=β^2-1\end{cases}\overset{(+)}{\Rightarrow}2(συνxσυνy+ημxημy)=α^2+β^2-2\ \Leftrightarrow$$

$$2συν(x-y)=α^2+β^2-2\ \Leftrightarrow\ συν(x-y)=\frac{α^2+β^2-2}{2}\quad(10)$$

$$(7)\ \underset{(10)}{\overset{(8),(9)}{\Rightarrow}}\ \frac{2\frac{2αβ}{α^2+β^2}}{\frac{β^2-α^2}{α^2+β^2}+\frac{α^2+β^2-2}{2}}=γ\ \Leftrightarrow\ \frac{\frac{4αβ}{α^2+β^2}}{\frac{2(β^2-α^2)+(α^2+β^2)(α^2+β^2-2)}{2(α^2+β^2)}}=γ\ \Leftrightarrow$$

$$\frac{8αβ}{2β^2-2α^2+(α^2+β^2)(α^2+β^2-2)}=γ\ \Leftrightarrow\ \frac{8αβ}{(α^2+β^2)^2-4α^2}=γ\ \Leftrightarrow$$

$$\boxed{\mathbf{γ\cdot[(α^2+β^2)^2-4α^2]=8αβ}}$$ ή

$$\boxed{\mathbf{γ\cdot(α^2+β^2-2α)\cdot(α^2+β^2+2α)=8αβ}}$$ ή

$$\boxed{\mathbf{γ\cdot(α^2+β^2)^2=4α\cdot(2β+αγ)}}$$