Κελάφας
ΦΡΟΝΤΙΣΤΗΡΙΑ

ΠΡΟΑΓΩΓΙΚΕΣ ΕΞΕΤΑΣΕΙΣ Γ΄ ΤΑΞΗΣ ΕΣΠΕΡΙΝΟΥ ΕΝΙΑΙΟΥ ΛΥΚΕΙΟΥ

ΤΕΤΑΡΤΗ 6 ΙΟΥΝΙΟΥ 2001

ΑΠΑΝΤΗΣΕΙΣ ΣΤΑ ΜΑΘΗΜΑΤΙΚΑ ΘΕΤΙΚΗΣ & ΤΕΧΝΟΛΟΓΙΚΗΣ ΚΑΤΕΥΘΥΝΣΗΣ

ΘΕΜΑ 1ο

Α. Α – 3, Β – 2, Γ – 1.

Β.1. α) Κ (3 , 10) και **ρ = 8**

β) $(11 - 3)^2 + (10 - 10)^2 = 64 \Leftrightarrow 8^2 + 0^2 = 64$ ισχύει
άρα **το σημείο Α (11 , 10) ανήκει στον κύκλο.**

Β.2. $\varepsilon : xx_1 + yy_1 = 8 \Rightarrow \varepsilon : 2x + 2y = 8 \Leftrightarrow$ $\boldsymbol{\varepsilon : x + y = 4}$

ΘΕΜΑ 2ο

1) $\vec{\alpha} \perp \vec{\beta} \Leftrightarrow \vec{\alpha}\cdot\vec{\beta} = 0 \Leftrightarrow 1\cdot 2 + (\lambda + 2)\cdot 1 = 0 \Leftrightarrow$
$2 + \lambda + 2 = 0 \Leftrightarrow \lambda + 4 = 0 \Leftrightarrow \boldsymbol{\lambda = -4}$

2) $\vec{\alpha} // \vec{\beta} \Leftrightarrow \det\left(\vec{\alpha}, \vec{\beta}\right) = 0 \Leftrightarrow \begin{vmatrix} 1 & \lambda + 2 \\ 2 & 1 \end{vmatrix} = 0 \Leftrightarrow$

$1\cdot 1 - 2(\lambda + 2) = 0 \Leftrightarrow 1 - 2\lambda - 4 = 0 \Leftrightarrow -2\lambda = 3 \Leftrightarrow \boldsymbol{\lambda = -\frac{3}{2}}$

3) $2\vec{\alpha} - \vec{\beta} = \vec{\gamma} \Rightarrow 2\cdot(1 , \lambda + 2) - (2 , 1) = (0 , 7) \Leftrightarrow$
$(2 , 2\lambda + 4) - (2 , 1) = (0 , 7) \Leftrightarrow (0 , 2\lambda + 3) = (0 , 7) \Rightarrow$
$2\lambda + 3 = 7 \Leftrightarrow 2\lambda = 4 \Leftrightarrow \boldsymbol{\lambda = 2}$

ΘΕΜΑ 3ο

α) $\left.\begin{aligned} x_K &= \frac{x_A + x_O}{2} = \frac{0+0}{2} = 0 \\ y_K &= \frac{y_A + y_O}{2} = \frac{12+0}{2} = 6 \end{aligned}\right\} \Rightarrow \mathbf{K\,(0\,,6)}$

$\left.\begin{aligned} x_\Lambda &= \frac{x_A + x_B}{2} = \frac{0+6}{2} = 3 \\ y_\Lambda &= \frac{y_A + y_B}{2} = \frac{12+8}{2} = 10 \end{aligned}\right\} \Rightarrow \mathbf{\Lambda\,(3\,,10)}$

β) $\overrightarrow{K\Lambda} = (x_\Lambda - x_K\ ,\ y_\Lambda - y_K) = (3 - 0\ ,\ 10 - 6) = (3\ ,\ 4)$

$|\overrightarrow{K\Lambda}| = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25}$, άρα $\mathbf{|\overrightarrow{K\Lambda}| = 5}$

γ) $\overrightarrow{OB} = (x_B - x_O\ ,\ y_B - y_O) = (6 - 0\ ,\ 8 - 0) = (6\ ,\ 8)$

$\lambda_{\overrightarrow{OB}} = \frac{8}{6} = \frac{4}{3}$

$(\varepsilon): y - y_A = \lambda_{\overrightarrow{OB}} \cdot (x - x_A) \Leftrightarrow$

$(\varepsilon): y - 12 = \frac{4}{3} \cdot (x - 0) \Leftrightarrow$

$(\varepsilon): y - 12 = \frac{4}{3}x \Leftrightarrow$

$\mathbf{(\varepsilon): y = \frac{4}{3}x + 12}$

ΘΕΜΑ 4ο

α) $\begin{cases} y = x + 5 \\ y = 10 \end{cases} \Leftrightarrow \begin{cases} 10 = x + 5 \\ y = 10 \end{cases} \Leftrightarrow \begin{cases} x = 5 \\ y = 10 \end{cases} \Rightarrow A\,(5\,,\,10)$

$\overrightarrow{BA} = (x_A - x_B\,,\,y_A - y_B) = (5 - 6\,,\,10 - 11) = (-1\,,\,-1)$

$\overrightarrow{B\Gamma} = (x_\Gamma - x_B\,,\,y_\Gamma - y_B) = (x - 6\,,\,10 - 11) = (x - 6\,,\,-1)$

$\overrightarrow{BA} \cdot \overrightarrow{B\Gamma} = 0 \Leftrightarrow -1\cdot(x - 6) + (-1)\cdot(-1) = 0 \Leftrightarrow -x + 6 + 1 = 0 \Leftrightarrow$

$7 - x = 0 \Leftrightarrow x = 7 \Rightarrow \mathbf{\Gamma\,(7\,,10)}$

β) $\overrightarrow{AB} = (x_B - x_A\,,\,y_B - y_A) = (6 - 5\,,\,11 - 10) = (1\,,\,1)$

$\overrightarrow{A\Delta} = (x_\Delta - x_A\,,\,y_\Delta - y_A) = (10 - 5\,,\,10 - 10) = (5\,,\,0)$

$\overrightarrow{AB} \cdot \overrightarrow{A\Delta} = 1\cdot 5 + 1\cdot 0 = 5 + 0 = 5$

$\left|\overrightarrow{AB}\right| = \sqrt{1^2 + 1^2} = \sqrt{1 + 1} = \sqrt{2}$

$\left|\overrightarrow{A\Delta}\right| = \sqrt{5^2 + 0^2} = \sqrt{25} = 5$

$\sigma\upsilon\nu\left(\overrightarrow{AB}\,,\,\overrightarrow{A\Delta}\right) = \dfrac{\overrightarrow{AB}\cdot\overrightarrow{A\Delta}}{\left|\overrightarrow{AB}\right|\cdot\left|\overrightarrow{A\Delta}\right|} = \dfrac{5}{\sqrt{2}\cdot 5} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2} = \sigma\upsilon\nu 45^0$

άρα $\left(\widehat{\overrightarrow{\mathbf{AB}}\,,\,\overrightarrow{\mathbf{A\Delta}}}\right) = \mathbf{45^0}$

γ) $\lambda_{\varepsilon_1} = 1$

$\varepsilon_3 \perp \varepsilon_1 \Rightarrow \lambda_{\varepsilon_3}\cdot\lambda_{\varepsilon_1} = -1 \Leftrightarrow \lambda_{\varepsilon_3}\cdot 1 = -1 \Leftrightarrow \lambda_{\varepsilon_3} = -1$

$(\varepsilon_3): y - y_\Delta = \lambda_{\varepsilon_3}\cdot(x - x_\Delta) \Leftrightarrow$

$(\varepsilon_3): y - 10 = -1\cdot(x - 10) \Leftrightarrow$

$(\varepsilon_3): y - 10 = -x + 10 \Leftrightarrow$

$\mathbf{(\varepsilon_3): y = -x + 20}$